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UNSTEADY DOWNWASH BEHIND A DELTA WING WITH SUPERSONIC MOTION, (U)
NOV 78 R S SOLOMONYAN

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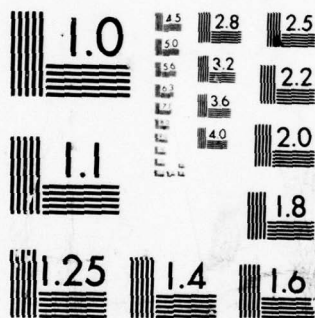


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UNSTEADY DOWNWASH BEHIND A DELTA WING WITH SUPERSONIC MOTION

By

R. Sh. Solomonyan



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UNSTEADY DOWNWASH BEHIND A DELTA WING WITH
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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З э	<i>З э</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

*ye initially, after vowels, and after ъ, ы; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh ⁻¹
cos	cos	ch	cosh	arc ch	cosh ⁻¹
tg	tan	th	tanh	arc th	tanh ⁻¹
ctg	cot	cth	coth	arc cth	coth ⁻¹
sec	sec	sch	sech	arc sch	sech ⁻¹
cosec	csc	csch	csch	arc csch	csch ⁻¹

Russian	English
rot	curl
lg	log

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UNSTEADY DOWNWASH BEHIND A DELTA WING WITH SUPERSONIC MOTION.

B. Sh. Solomonyan

The question of determining the aerodynamic characteristics of the tail assembly of flight vehicle has great practical value both from the point of view of control of flight vehicles and from the point of view of aeroelasticity.

During the determination of the aerodynamic forces and tail moments, arises the question of the determination of wing downwash. This task in special cases is examined in works [5, 7], etc., but common/general/total setting is given by N. N. Kislyagin [3].

In this article are given the formulas for calculating the downwash upon the common/general/total formulation of the problem for the delta wing, which has supersonic leading edges.

Let the fine/thin slightly-curved delta wing move in ideal

compressible liquid with low angle of attack and with certain angle of slip β_0 . Let us consider that the basic motion of wing is rectilinear forward/progressive with the constant supersonic velocity U . Let us assume also that, besides basic motion, the wing completes small additional oscillation/vibrations.

Downwash is represented through the coefficients of the rotary derivatives [2, 3] and we will use the formulas of the calculation of these coefficients [2], which for the case of small Strouhal numbers take the form:

$$\begin{aligned} \theta_i^{(n)}(x, y) = \frac{1}{\pi^2} \left\{ -\frac{1}{2} V. p. \int_{\chi(x)}^{\chi} \frac{f_i^{(n)}[\bar{\chi}(\eta), \eta] d\eta}{V \sqrt{y-\eta} \sqrt{x-\bar{\chi}(\eta)}} + \right. \\ \left. + \int_{\chi(x)}^{\chi} \int_{\chi(\eta)}^{\chi} \frac{1}{V(x-\xi)(y-\eta)} \frac{\partial^2}{\partial \xi \partial \eta} f_i^{(n)}(\xi, \eta) d\xi d\eta \right\} \quad (1) \\ (i=1, 2; n=1, 3, 4) \end{aligned}$$

Formula (1) is written in the dimensionless characteristic coordinates xy , which are connected with the Cartesian coordinates $x_1 y_1$ (Fig. 1) as follows:

$$x = \frac{2}{lk} x_1 - \frac{2}{l} y_1, \quad y = \frac{1}{lk} x_1 + \frac{2}{l} y_1. \quad (2)$$

where $k = \sqrt{M^2 - 1}$, $M = U/a$ - Mach number, a - the speed of sound in the undisturbed flow, l - characteristic linear dimension (spread/scope) of wing.

In formula (1) sign $V.p. \int$ indicates the principal value of integral according to Hadamard [4], $y = \gamma(x)$ there is an equation of trailing wing edge, $x = \gamma(y)$ - the equation of the same wing edge, solved relatively by the variable x . Functions

$f_v^{(i)}(x, y)$ ($i=1, 2$; $v=1, 3, 4$) are expressed by the formulas

$$f_v^{(1)}(x, y) = - \int \int_{x_0+y_0}^x \frac{B_v^{(1)}(\xi, \eta) d\xi d\eta}{V(x-\xi)(y-\eta)} + A_v^{(1)}(x, y) \quad (3)$$

$$f_v^{(2)}(x, y) = \frac{\lambda}{8} \left(k + \frac{1}{k} \right) \int \int_{x_0+y_0}^x B_v^{(1)}(\xi, \eta) \frac{x-\xi+y-\eta}{V(x-\xi)(y-\eta)} d\xi d\eta + A_v^{(2)}(x, y), \quad (v=1, 3, 4) \quad (4)$$

where $B_v^{(1)}(x, y)$ they are assigned by the condition of steady flow around of the wing and have following values [2]:

$$B_1^{(1)}(x, y) = -1; \quad B_3^{(1)}(x, y) = -\frac{\lambda}{8}(x-y); \quad B_4^{(1)}(x, y) = -\frac{\lambda k}{8}(x+y)$$

$\lambda = l^2/S$ - aspect ratio, S - wing area.

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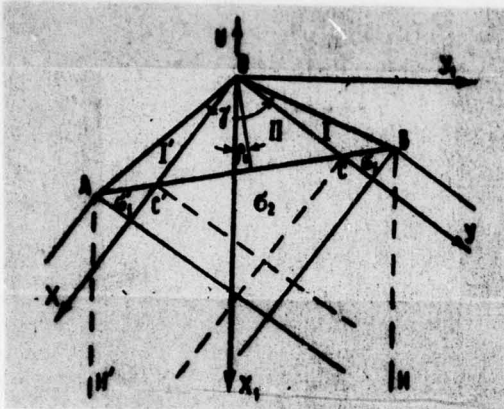


Fig. 1.

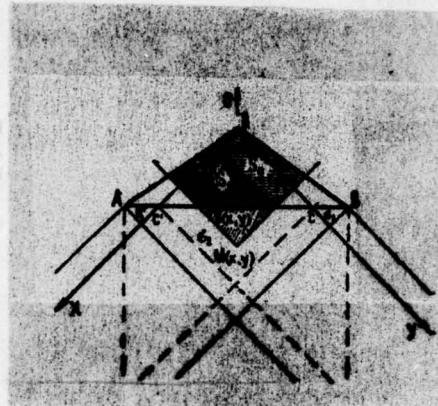


Fig. 2.

The equations of wing edges will be (Fig. 1):

$$y = -a_0^2 x - \text{by rightist front/leading of edge,}$$

$$y = -a_1^2 x - \text{by left front/leading of edge,}$$

$$y = -\beta^2 x + e = \chi(x) - \text{a trailing edge,}$$

where angular coefficients and number e are expressed as the geometric parameters of wing and the Mach number

$$\begin{aligned} a_0^2 &= -\frac{1 + k \operatorname{tg}(\gamma - \beta_0)}{1 - k \operatorname{tg}(\gamma - \beta_0)}, & a_1^2 &= -\frac{1 - k \operatorname{tg}(\gamma + \beta_0)}{1 + k \operatorname{tg}(\gamma + \beta_0)} \\ \beta^2 &= -\frac{1 + k \operatorname{ctg} \beta_0}{1 - k \operatorname{ctg} \beta_0}, & e &= -\frac{8\sqrt{1 + \operatorname{ctg}^2 \beta_0}}{\lambda(1 - k \operatorname{ctg} \beta_0)} \end{aligned}$$

By the interference waves, which proceed from point O, wing is divided into three regions (I, II and I'), but trailing edge - to three cuts BC, CC' and C'A with different analytical expressions for potentials.

In formulas (3) and (4) functions $A_n^{(i)}(x, y)$ are the values of the potentials of the disturbed velocities on the cuts indicated. For calculating these functions with small Strouhal numbers, we use the formulas, available in work [2].

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For a delta wing when point M is arranged/located in region σ_2 (Fig. 2), functions $f_i^{(n)}(x, y)$ have the following expressions:

$$f_1^{(n)}(x, y) = -\frac{\pi}{\beta} Z_2(x, y) + \sum_{j=0}^1 g_{1,j}^{(n,1)} \left\{ Z_j \left[\frac{\pi}{2} + (-1)^j \psi_j \right] - \frac{Q_{1j}}{\gamma} \left[\frac{\pi}{2} + (-1)^j \bar{\psi}_j \right] \right\} \quad (5)$$

$$f_3^{(n)}(x, y) = -\frac{\lambda \pi}{8} \left[\frac{e - \gamma x}{\beta} Z_2(x, y) - \frac{1 + 3\beta^2}{4\beta^3} Z_2^2(x, y) \right] + \\ + \sum_{j=0}^1 \sum_{n=0}^1 g_{3,j}^{(1,2-n)} \left\{ Z_j^{2-n} y^n \left[\frac{\pi}{2} + (-1)^j \psi_j \right] - \frac{1}{\gamma^2} Q_{1,j}^{(0)} Q_{1,j}^{2-n} \left[\frac{\pi}{2} + (-1)^j \bar{\psi}_j \right] \right\} + \\ + \sum_{n=0}^1 [b_{n,1-n}^{(1)} x^{n+\frac{1}{2}} y^{\frac{3}{2}-n} + p_n^{(1)} (y-x)^n R(x, y)] \quad (6)$$

$$f_4^{(n)}(x, y) = \frac{\lambda k \pi}{8} \left[\frac{3\beta^2 - 1}{4\beta^3} Z_2^2(x, y) + \frac{e + (1 - \beta^2)x}{\beta} Z_2(x, y) \right] + \\ + \sum_{j=0}^1 \sum_{n=0}^1 g_{4,j}^{(1,2-n)} \left\{ Z_j^{2-n} y^n \left[\frac{\pi}{2} + (-1)^j \psi_j \right] - \frac{1}{\gamma^2} Q_{1,j}^{(0)} Q_{1,j}^{2-n} \left[\frac{\pi}{2} + (-1)^j \bar{\psi}_j \right] \right\} + \\ + \sum_{n=0}^1 [a_{n,1-n}^{(1)} x^{n+\frac{1}{2}} y^{\frac{3}{2}-n} + q_n^{(1)} (y-x)^n R(x, y)] \quad (7)$$

$$f_5^{(n)}(x, y) = -\frac{\lambda M^2 (1 - \beta^2) \pi}{32 \beta^4} Z_2^2(x, y) +$$

$$+ \sum_{j=0}^1 g_{1,j}^{(2,2)} \left\{ Z_j^2 \left[\frac{\pi}{2} + (-1)^j \psi_j \right] - \frac{Q_{1,j}^2}{v^2} \left[\frac{\pi}{2} + (-1)^j \bar{\psi}_j \right] \right\} +$$

$$+ \sum_{n=0}^1 [a_{n,1-n}^{(2)} x^{n+\frac{1}{2}} y^{\frac{3}{2}-n} + r_n^{(2)} (y-x)^n R(x, y)] \quad (8)$$

$$f_3^{(2)}(x, y) = \frac{\lambda^2 \pi}{64} \left(k + \frac{1}{k} \right) \left\{ \frac{\nabla}{4\beta^3} (e - \nabla x) Z_2^2(x, y) + \frac{\nabla^2}{8\beta^3} Z_2^2(x, y) \right\} +$$

$$+ \sum_{j=0}^1 \sum_{n=0}^1 g_{3,j}^{(2,3-n)} \left\{ Z_j^{3-n} y^n \left[\frac{\pi}{2} + (-1)^j \psi_j \right] - \right.$$

$$\left. - \frac{Q_1^n(0) Q_1^{3-n}}{v^3} \left[\frac{\pi}{2} + (-1)^j \bar{\psi}_j \right] \right\} +$$

$$+ \sum_{n=0}^2 [b_{n,2-n}^{(2)} x^{n+\frac{1}{2}} y^{\frac{5}{2}-n} + p_n^{(2)} (y-x)^n R(x, y)] \quad (9)$$

$$f_4^{(2)}(x, y) = \frac{\lambda^2 k \pi}{64} \left(k + \frac{1}{k} \right) \left\{ \frac{\nabla}{4\beta^3} [(\beta^2 - 1)x - e] Z_2^2(x, y) + \right.$$

$$\left. + \frac{1}{24\beta^3} (9 + 4\beta^2 - 9\beta^4) Z_2^3(x, y) \right\} +$$

$$+ \sum_{j=0}^1 \sum_{n=0}^1 g_{4,j}^{(2,3-n)} \left\{ Z_j^{3-n} y^n \left[\frac{\pi}{2} + (-1)^j \psi_j \right] - \right.$$

$$\left. - \frac{Q_{1,j}^{3-n} Q_1^n(0)}{v^3} \left[\frac{\pi}{2} + (-1)^j \bar{\psi}_j \right] \right\} +$$

$$+ \sum_{n=0}^2 [q_{n,2-n}^{(2)} x^{n+\frac{1}{2}} y^{\frac{5}{2}-n} + q_n^{(2)} (y-x)^n R(x, y)] \quad (10)$$

In formulas (5)-(10) and for further calculations, for purpose of a reduction in the recording, are introduced the following designations of the functions

$$Z_j = y + \alpha_j^2 x, \quad Z_1(x, y) = y + \beta^2 x - e, \quad Z_2 = y - \beta^2 x + e$$

$$Q_{0,j} = Q_0(y, \alpha_j) = \Delta_j y + \alpha_j^2 e, \quad \bar{\chi}(y) = \frac{1}{\beta^2} (e - y)$$

$$Q_{1,j} = Q_1(x, y, \alpha_j) = \Delta_j (y - x) + (1 + \alpha_j^2) e$$

$$Q_1(0) = Q_1(x, y, 0) = \beta^2 (y - x) + e$$

$$Q_{2,j} = Q_2(x, \alpha_j) = -\Delta_j x + e$$

$$Q_{3,j} = Q_3(x, \alpha_j) = -\Delta_j x + (\alpha_j^2 + \beta^2) e$$

$$\Omega_j = \Omega(x, \alpha_j) = \alpha_j^2 (37\beta^2 x + 14e)$$

$$\psi_{1,j} = \psi_1(x, y, \alpha_j) = \arcsin \frac{y - \alpha_j^2 x}{y + \alpha_j^2 x}, \quad \psi_{0,j} = \bar{\psi}_{0,j} = \frac{\pi}{2}$$

$$\bar{\psi}_{1,j} = \bar{\psi}_1(x, y, \alpha_j) = \arcsin \frac{(\beta^2 + \alpha_j^2)(y - x) + (1 - \alpha_j^2) e}{\Delta_j (y - x) + (1 + \alpha_j^2) e}$$

$$C(y, \alpha_j) = \arcsin \frac{(\alpha_j^2 + \beta^2) y - \alpha_j^2 e}{\Delta_j y + \alpha_j^2 e}$$

$$R(x, y) = \sqrt{e^2 + (\beta^2 - 1) e (y - x) - \beta^2 (y - x)^2}$$

$$R_1(x, y, \alpha_j) = \arctg \sqrt{\frac{y(y + \beta^2 x - e)}{(e - y)(y + \alpha_j^2 x)}}$$

$$R_2(x, y) = \arctg \sqrt{\frac{y + \beta^2 x - e}{e - y}}$$

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Are given below the values of coefficients, which participate in the calculations which depend only on the geometric wing characteristics (α_0 , α_1 , β , λ) and of Mach number.

$$\begin{aligned} \gamma_1 &= \alpha_0^2 \alpha_1^2, \quad \gamma_2 = \alpha_0^2 - \alpha_1^2, \quad \gamma_3 = \alpha_0^2 + \alpha_1^2, \quad \Delta_j = \beta^2 - \alpha_j^2, \quad v = 1 + \beta^2 \\ g_{1,j}^{(1,1)} &= \frac{1}{\alpha_j}, \quad g_{1,j}^{(2,2)} = -\frac{\lambda M^2}{32k} \frac{1 + \alpha_j^2}{\alpha_j^3}, \quad g_{3,j}^{(1,1)} = -\frac{\lambda(1 + \alpha_j^2)}{8\alpha_j^3} \\ g_{3,j}^{(1,2)} &= \frac{\lambda(3 + \alpha_j^2)}{32\alpha_j^3}, \quad g_{3,j}^{(2,2)} = \frac{\lambda^2 M^2}{256k} \frac{(1 + \alpha_j^2)^2}{\alpha_j^5}, \quad g_{3,j}^{(2,3)} = -\frac{\lambda^2 M^2 (1 + \alpha_j^2)^2}{512k \alpha_j^5} \\ g_{4,j}^{(1,1)} &= -\frac{\lambda k (1 - \alpha_j^2)}{8\alpha_j^3}, \quad g_{4,j}^{(1,2)} = \frac{\lambda k}{32} \frac{3 - \alpha_j^2}{\alpha_j^3}, \quad g_{4,j}^{(2,2)} = \frac{\lambda^2 M^2}{256} \frac{\alpha_j^4 - 1}{\alpha_j^5} \\ g_{4,j}^{(2,3)} &= -\frac{\lambda^2 M^2 (3\alpha_j^4 - 4\alpha_j^2 - 3)}{1536 \alpha_j^5}, \quad (j = 0, 1) \\ a_{1,0}^{(2)} &= \frac{\gamma_2 M^2 \lambda}{16k}, \quad a_{0,1}^{(2)} = \frac{\lambda \gamma_2 M^2}{16 \gamma_1 k}, \quad b_{1,0}^{(1)} = \frac{\lambda \gamma_2}{16}, \quad b_{0,1}^{(1)} = -\frac{\lambda \gamma_2}{16 \gamma_1} \\ b_{2,0}^{(2)} &= \frac{\lambda^2 M^2}{256k} \gamma_2 (2 + \gamma_3), \quad b_{1,1}^{(2)} = -\frac{\lambda^2 M^2 \gamma_2}{384 k \gamma_1} (1 - \gamma_1) \\ b_{0,2}^{(2)} &= -\frac{\lambda^2 M^2 \gamma_2}{256 k \gamma_1^2} (\gamma_3 + 2\gamma_1), \quad d_{1,0}^{(1)} = \frac{\lambda k \gamma_2}{16}, \quad d_{0,1}^{(1)} = \frac{\lambda k \gamma_2}{16 \gamma_1} \\ d_{2,0}^{(2)} &= \frac{\lambda^2 M^2}{768} \gamma_2 (4 - 3\gamma_3), \quad d_{1,1}^{(2)} = -\frac{\lambda^2 M^2 \gamma_2}{384 \gamma_1} (1 + \gamma_1) \\ d_{0,2}^{(2)} &= \frac{\lambda^2 M^2 \gamma_2}{768 \gamma_1^2} (4\gamma_1 - 3\gamma_3), \quad p_0^{(1)} = \frac{\lambda \gamma_2}{16 \gamma^2 \gamma_1} (1 - \gamma_1) e \\ p_1^{(1)} &= \frac{\lambda \gamma_2 (\beta^2 + \gamma_1)}{16 \gamma^2 \gamma_1} \end{aligned}$$

$$\begin{aligned}
p_0^{(2)} &= \frac{\lambda^2 \gamma_1 M^2}{768 \gamma_1^2 k \nabla^3} [-3\gamma_1^2(2 + \gamma_1) + 2\gamma_1(1 - \gamma_1) + 3(2\gamma_1 + \gamma_1)] e^2 \\
p_1^{(2)} &= -\frac{\lambda^2 M^2 \gamma_1}{768 k \gamma_1^2 \nabla^3} [6\gamma_1^2(2 + \gamma_1) + 2(\beta^2 - 1)(1 - \gamma_1)\gamma_1 + 6\beta^2(2\gamma_1 + \gamma_1)] e \\
p_2^{(2)} &= -\frac{\lambda^2 M^2 \gamma_1}{768 k \gamma_1^2 \nabla^3} [3\gamma_1^2(2 + \gamma_1) + 2\beta^2\gamma_1(1 - \gamma_1) - 3(2\gamma_1 + \gamma_1)\beta^2] \\
q_0^{(1)} &= \frac{\lambda k \gamma_1}{16 \gamma_1 \nabla^2} (1 + \gamma_1) e, \quad q_1^{(1)} = \frac{\lambda k \gamma_1}{16 \gamma_1 \nabla^2} (\beta^2 - \gamma_1) \\
q_0^{(2)} &= -\frac{\lambda^2 M^2 \gamma_1}{768 \gamma_1^2 \nabla^3} [\gamma_1^2(4 - 3\gamma_1) - 2\gamma_1(1 + \gamma_1) + (4\gamma_1 - 3\gamma_1)] e^2 \\
q_1^{(2)} &= \frac{\lambda^2 M^2 \gamma_1}{768 \gamma_1^2 \nabla^3} [2\gamma_1^2(4 - 3\gamma_1) - 2\gamma_1(1 + \gamma_1)(1 - \beta^2) - 2\beta^2(4\gamma_1 - 3\gamma_1)] e \\
q_2^{(2)} &= -\frac{\lambda^2 M^2 \gamma_1}{768 \gamma_1^2 \nabla^3} [\gamma_1^2(4 - 3\gamma_1) + 2\beta^2\gamma_1(1 + \gamma_1) + \beta^2(4\gamma_1 - 3\gamma_1)] \\
r_0^{(2)} &= -\frac{\lambda M^2}{16k} \gamma_1 \frac{1 + \gamma_1}{\nabla^2 \gamma_1} e, \quad r_1^{(2)} = -\frac{\lambda M^2}{16k} \gamma_1 \frac{\beta^2 - \gamma_1}{\nabla^2 \gamma_1} \\
\omega_j &= 3e\beta^2 - 14a_j^2
\end{aligned}$$

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Substituting in formula (2) of the value of functions $f_i^{(n)}(x, y)$ from (5)-(10) and performing integration, we will obtain expressions for $\theta_i^{(n)}(x, y)$ in the following form:

$$\begin{aligned}
\theta_1^{(1)}(x, y) &= -1 + \frac{1}{\pi^2} \sum_{j=0}^1 g_{1,j}^{(1,1)} \left\{ F_{0,j}^{(0,1)}(x, y) + (-1)^j F_{1,j}^{(0,1)}(x, y) - \right. \\
&\quad \left. - \frac{1}{\nabla} [L_{0,j}^{(0,1)}(x, y) + (-1)^j L_{1,j}^{(0,1)}(x, y)] \right\} \quad (11)
\end{aligned}$$

$$\begin{aligned}
\theta_3^{(1)}(x, y) &= \frac{\lambda}{8} \left[\frac{1 + 3\beta^2}{2\beta^2} Z_2(x, y) + (e - \nabla x) + \right. \\
&+ \frac{1}{\pi^2} \sum_{j=0}^1 \sum_{n=0}^1 g_{3,j}^{(1,2-n)} \left\{ F_{0,j}^{(n,2-n)}(x, y) + (-1)^j F_{1,j}^{(n,2-n)}(x, y) - \right. \\
&\quad \left. - \frac{1}{\nabla^2} [L_{0,j}^{(n,2-n)}(x, y) + (-1)^j L_{1,j}^{(n,2-n)}(x, y)] \right\} + \\
&+ \frac{1}{\pi^2} \sum_{n=0}^1 [b_{n,1-n}^{(1)} H_{n,1-n}(x, y) + p_n^{(1)} N_n(x, y)] \quad (12)
\end{aligned}$$

$$\theta_4^{(1)}(x, y) = -\frac{\lambda k}{8} \left\{ \frac{3\beta^2 - 1}{4\beta^2} Z_2(x, y) + (1 - \beta^2)x + e \right\} +$$

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$$\begin{aligned}
& + \frac{1}{\pi^2} \sum_{j=0}^1 \sum_{n=0}^1 g_{1,j}^{(1,2-n)} \left\{ F_{0,j}^{(n,2-n)}(x,y) + (-1)^j F_{1,j}^{(n,2-n)}(x,y) - \right. \\
& \quad \left. - \frac{1}{\nabla^2} [L_{0,j}^{(n,2-n)}(x,y) + (-1)^j L_{1,j}^{(n,2-n)}(x,y)] \right\} + \\
& \quad + \frac{1}{\pi^2} \sum_{n=0}^1 [a_{n,1-n}^{(1)} H_{n,1-n}(x,y) + q_n^{(1)} N_n(x,y)] \quad (13)
\end{aligned}$$

$$\begin{aligned}
\theta_1^{(2)}(x,y) &= - \frac{\lambda M^2 (1-\beta^2)}{32 \beta^4 k} Z_2^2(x,y) + \\
& + \frac{1}{\pi^2} \sum_{j=0}^1 g_{1,j}^{(2,2)} \left\{ F_{0,j}^{(0,2)}(x,y) + (-1)^j F_{1,j}^{(0,2)}(x,y) - \right. \\
& \quad \left. - \frac{1}{\nabla^2} [L_{0,j}^{(0,2)}(x,y) + (-1)^j L_{1,j}^{(0,2)}(x,y)] \right\} + \\
& \quad + \frac{1}{\pi^2} \sum_{n=0}^1 [a_{n,1-n}^{(2)} H_{n,1-n}(x,y) + r_n^{(2)} N_n(x,y)] \quad (14)
\end{aligned}$$

$$\begin{aligned}
\theta_3^{(2)}(x,y) &= \frac{\lambda}{8} \left[\frac{\nabla}{2\beta^2} (e - \nabla x) Z_2(x,y) + \frac{3}{8} \frac{\nabla^2}{\beta^4} Z_2^2(x,y) \right] + \\
& + \frac{1}{\pi^2} \sum_{j=0}^1 \sum_{n=0}^1 g_{3,j}^{(2,3-n)} \left\{ F_{0,j}^{(n,3-n)}(x,y) + (-1)^j F_{1,j}^{(n,3-n)}(x,y) - \right. \\
& \quad \left. - \frac{1}{\nabla^2} [L_{0,j}^{(n,3-n)}(x,y) + (-1)^j L_{1,j}^{(n,3-n)}(x,y)] \right\} + \\
& \quad + \frac{1}{\pi^2} \sum_{n=0}^2 [b_{n,2-n}^{(2)} H_{n,2-n}(x,y) + p_n^{(2)} N_n(x,y)] \quad (15)
\end{aligned}$$

$$\begin{aligned}
\theta_4^{(2)}(x,y) &= \frac{\lambda k}{8} \left\{ \frac{\nabla}{2\beta^2} [(\beta^2 - 1)x - e] Z_2(x,y) + \right. \\
& \quad \left. + \frac{1}{8\beta^4} (9 + 4\beta^2 - 9\beta^4) Z_2^2(x,y) \right\} + \\
& + \frac{1}{\pi^2} \sum_{j=0}^1 \sum_{n=0}^1 g_{4,j}^{(2,3-n)} \left\{ F_{0,j}^{(n,3-n)}(x,y) + (-1)^j F_{1,j}^{(n,3-n)}(x,y) - \right. \\
& \quad \left. - \frac{1}{\nabla^2} [L_{0,j}^{(n,3-n)}(x,y) + (-1)^j L_{1,j}^{(n,3-n)}(x,y)] \right\} + \\
& \quad + \frac{1}{\pi^2} \sum_{n=0}^2 [a_{n,2-n}^{(2)} H_{n,2-n}(x,y) + q_n^{(2)} N_n(x,y)] \quad (16)
\end{aligned}$$

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In formulas (11) - (16) for downwashes $\theta_i^{(n)}(x, y)$ enter the functions

$$F_{l,j}^{(n,m)}(x, y) = \frac{\partial}{\partial y} \int_{\chi(x)}^y \frac{d\eta}{V y - \eta} \frac{\partial}{\partial x} \int_{\bar{\chi}(\eta)}^x \frac{\eta^n Z_l^m}{V x - \xi} \phi_l(\xi, \eta, \alpha_j) d\xi$$

$$(n = 0, 1; m = 1, 2, 3; l = 0, 1)$$

$$L_{l,j}^{(n,m)}(x, y) = \frac{\partial}{\partial y} \int_{\chi(x)}^y \frac{d\eta}{V y - \eta} \frac{\partial}{\partial x} \times \\ \times \int_{\bar{\chi}(\eta)}^x \frac{Q_l^n(\xi, \eta, 0) Q_l^m(\xi, \eta, \alpha_j)}{V x - \xi} \phi_l(\xi, \eta, \alpha_j) d\xi$$

$$H_{n,m}(x, y) = \frac{\partial}{\partial y} \int_{\chi(x)}^y \frac{\eta^m V \sqrt{\eta} d\eta}{V y - \eta} \frac{\partial}{\partial x} \int_{\bar{\chi}(\eta)}^x \frac{\xi^n V \sqrt{\xi} d\xi}{V x - \xi}, \quad (n, m = 0, 1, 2)$$

$$N_n(x, y) = \frac{\partial}{\partial y} \int_{\chi(x)}^y \frac{d\eta}{V y - \eta} \frac{\partial}{\partial x} \int_{\bar{\chi}(\eta)}^x \frac{(\eta - \xi)^n R(\xi, \eta) d\xi}{V x - \xi}$$

which, after the execution of actions in right sides, take the form

$$F_{0,j}^{(0,1)}(x, y, \alpha_j) = \frac{\pi^2 (\alpha_j^2 + \beta^2)}{4 \beta \alpha_j} \quad (17)$$

$$F_{0,j}^{(0,2)}(x, y, \alpha_j) = \frac{\pi^2}{8 \beta^3} \{ [4(2\beta^2 + \Delta_j) \alpha_j^2 - 3\Delta_j^2] y + \\ + \beta^2 [\Delta_j^2 + 4\alpha_j^2(\beta^2 + \alpha_j^2)] x - \Delta_j^2 e \} \quad (18)$$

$$F_{0,j}^{(0,3)}(x, y, \alpha_j) = \frac{3\pi^2}{32 \beta^5} \{ -\Delta_j^3 (4y^2 + Z_3^2) - 4\alpha_j^2 \Delta_j^2 e (2y + \\ + Z_3) + 8\alpha_j^4 \Delta_j e^2 + 6\alpha_j^2 Q_{0,j}^2 + 4\alpha_j^2 \beta^2 Q_{0,j} Q_{2,j} + \\ + 6\alpha_j^2 \beta^4 Q_{2,j}^2 + 24\alpha_j^2 \beta^2 Z_2(x, y)(\beta^2 Q_{2,j} + 3Q_{0,j}) + 16\alpha_j^4 \beta^2 Z_2^2(x, y) \} \quad (19)$$

$$F_{0,j}^{(1,1)}(x, y, \alpha_j) = -\frac{\pi^2}{8 \beta} (3\Delta_j y - \beta^2 \Delta_j x + \beta^2 e - 3\alpha_j^2 e) + \\ + \frac{\pi^2 \alpha_j^2}{4 \beta} [2y + Z_1(x, y)] \quad (20)$$

$$F_{0,j}^{(1,2)}(x, y, \alpha_j) = -\frac{3\pi^2 \Delta_j}{32 \beta^3} (4y^2 + Z_3^2) + \frac{\pi^2 \alpha_j^2}{4 \beta^3} Z_2(x, y) \times \\ \times [(\alpha_j^2 - 4\Delta_j) \beta^2 x + (5\beta^2 + \Delta_j) y + (\beta^2 + 2\alpha_j^2) e] +$$

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$$+ \frac{\pi^2 \alpha_j^2}{4\beta^3} [4\beta^2 (\Delta_j x - e) x + 5\alpha_j^2 e^2] \quad (21)$$

$$L_{0,j}^{(0,1)}(x, y, \alpha_j) = -\frac{\Delta_j \pi^2}{4\alpha_j \beta} + \frac{\pi^2 \Delta_j}{2\gamma \alpha_j \beta} (y + \beta^2 x - e) \quad (22)$$

$$L_{0,j}^{(0,2)}(x, y, \alpha_j) = -\frac{\pi^2 \Delta_j}{8\beta^3} [\Delta_j (3\gamma^2 + 12\gamma - 8) y - \Delta_j (\gamma^2 - 4\gamma + 8) \beta^2 x + [\gamma^2 (\beta^2 - 5\alpha_j^2) + 4\beta^2 (1 - \beta^2) + 4\alpha_j^2 (1 + 3\beta^2)] e] \quad (23)$$

$$L_{0,j}^{(0,3)}(x, y) = -\frac{3\pi^2 \gamma^2 \Delta_j}{32\beta^5} \{ \Delta_j^2 (4y^2 + Z_3^2) + 4\alpha_j^2 \Delta_j e (2y + Z_3) - 8\alpha_j^4 e^2 \} - \frac{3\pi^2}{16\beta^5} \Delta_j [8\beta^4 Q_0^2(y, \alpha_j) + 2\beta^2 (3 + 2\beta^2) \Delta_j Q_0(y, \alpha_j) \times \\ \times Z_2(x, y) + \Delta_j^2 (3\beta^4 + 4\beta^2 + 3) Z_2^2(x, y) + 8\beta^4 Q_0(y, \alpha_j) Q_2(x, \alpha_j) + 8\beta^4 \gamma Q_2^2(x, \alpha_j) + 2\beta^2 (2\gamma - 1)(\gamma + 3) \Delta_j Q_2(x, \alpha_j) Z_2(x, y)] \quad (24)$$

$$L_{0,j}^{(1,1)}(x, y) = -\frac{\pi^2 \gamma^2}{8\beta} [\Delta_j (2y + Z_3) - 2\alpha_j^2 e] - \frac{\pi^2}{4\beta} Z_2(x, y) \times \\ \times [(3\gamma - 2) \Delta_j y - \Delta_j \gamma \beta^2 x + (\beta^4 + \alpha_j^2 \beta^2 + \Delta_j) e] \quad (25)$$

$$L_{0,j}^{(1,2)}(x, y) = -\frac{\pi^2 \gamma^2}{32\beta^3} \{ 3\Delta_j^2 [4y^2 + Z_3^2] + 8\Delta_j \alpha_j^2 e [2y + Z_3] - 8\alpha_j^4 e^2 \} - \frac{3\pi^2}{16\beta^3} \Delta_j^2 \gamma (4 - \gamma) Z_2^2(x, y) - \frac{\pi^2}{2\beta^3} [\Delta_j (2\gamma - 1) \times \\ \times [3\beta^2 Q_{1,j} - \alpha_j^2 \gamma e] Z_2(x, y) + 3\beta^2 [Q_0(y, \alpha_j) - \Delta_j Z_2(x, y)] Q_0(y, \alpha_j) + 3\beta^4 [Q_{1,j} + \beta^2 Q_3(x, \alpha_j)] Q_2(x, \alpha_j) - \alpha_j^2 e \gamma [Q_{1,j} + \gamma Q_2(x, \alpha_j)]] \quad (26)$$

$$F_{1,j}^{(0,0)}(x, y, \alpha_j) = -\frac{1}{2\beta} V. p. \int_{\chi(x)}^{\gamma} \frac{Q_0(\eta, \alpha_j) C(\eta, \alpha_j) d\eta}{V y - \eta^3 V \eta + \beta^2 x - e} - J_1 + \\ + \frac{\alpha_j^2}{\beta} \int_{\chi(x)}^{\gamma} \frac{C(\eta, \alpha_j) d\eta}{V y - \eta V Z_2(x, \eta)} - \quad (27)$$

$$- 2\alpha_j \int_{\chi(x)}^{\gamma} \frac{R_1(x, \eta, \alpha_j) d\eta}{V y - \eta V \eta + \alpha_j^2 x} + \alpha_j \int_{\chi(x)}^{\gamma} \frac{R_2(x, \eta) d\eta}{V \eta (y - \eta)}$$

$$F_{1,j}^{(0,2)}(x, y, \alpha_j) = -\frac{1}{2\beta^3} V. p. \int_{\chi(x)}^{\gamma} \frac{Q_0^2(\eta, \alpha_j) C(\eta, \alpha_j) d\eta}{V y - \eta^3 V \eta + \beta^2 x - e} - \\ - \frac{1}{2\beta^3} [(\alpha_j^2 + 2\beta^2) J_1 - \alpha_j^2 [3\chi(x) + e] J_1 + 3\alpha_j^2 e \chi(x) J_0] +$$

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$$+ \frac{2a_j^2}{\beta^2} \int_{\chi(x)}^y [Q_0(\eta, a_j) + 2\beta^2 Z_2(x, \eta)] \frac{C(\eta, a_j) d\eta}{V y - \eta V Z_2(x, \eta)} + \quad (28)$$

$$+ \frac{a_j}{2} \int_{\chi(x)}^y \frac{10\eta + 3a_j^2 x}{V(y - \eta)\eta} R_2(x, \eta) d\eta -$$

$$- 8a_j \int_{\chi(x)}^y \sqrt{\frac{\eta + a_j^2 x}{y - \eta}} R_1(x, \eta, a_j) d\eta$$

$$F_{1,j}^{(0,3)}(x, y, a_j) = -\frac{1}{2\beta^2} V.p. \int_{\chi(x)}^y \frac{Q_0^3(\eta, a_j) C(\eta, a_j) d\eta}{V y - \eta^3 V Z_2(x, \eta)} +$$

$$+ \frac{a_j^2}{\beta^2} \int_{\chi(x)}^y [3Q_0^2(\eta, a_j) + 12\beta^2 Q_0(\eta, a_j) Z_2(x, \eta) +$$

$$+ 8a_j^2 \beta^2 Z_2^2(x, \eta)] \frac{C(\eta, a_j) d\eta}{V y - \eta V Z_2(x, \eta)} +$$

$$+ \frac{a_j}{80} \int_{\chi(x)}^y [880\eta^2 + 960a_j^2 x \eta + 165a_j^4 x^2] \frac{R_2(x, \eta)}{V \eta (y - \eta)} d\eta -$$

$$- 16a_j \int_{\chi(x)}^y \sqrt{\frac{(\eta + a_j^2 x)^3}{y - \eta}} R_1(x, \eta) d\eta + \frac{2e a_j^3}{\beta^4} [(3\beta^2 + a_j^2) J_2 +$$

$$+ [3a_j^2 e - (3\beta^2 + 5a_j^2) \chi(x)] J_1 - a_j^2 \chi(x) (4\beta^2 x - e) J_0] +$$

$$+ \frac{a_j^3}{80\beta^4} [-5\omega_j J_3 + [4\omega_j (2e - \beta^2 x) - 3\Omega_j(x)] J_2 - [3\omega_j e \chi(x) -$$

$$- 2\Omega_j(x)(2e - \beta^2 x)] J_1 - e \Omega_j(x) \chi(x) J_0] +$$

$$+ \frac{a_j}{80} [176 J_3 + 320a_j^2 x J_2 + 165a_j^4 x^2 J_1] - \frac{16}{5} a_j [J_3 +$$

$$+ 2a_j^2 x J_2 + a_j^4 x^2 J_1] - \frac{16a_j^3 e}{5\beta^4} [(a_j^2 + \beta^2) J_2 + [a_j^2 \beta^2 x -$$

$$- (\beta^2 + 2a_j^2) \chi(x)] J_1 - a_j^2 (2\beta^2 x - e) \chi(x) J_0] \quad (29)$$

$$F_{1,j}^{(1,1)}(x, y, a_j) = -\frac{1}{2\beta} V.p. \int_{\chi(x)}^y \frac{\eta Q_0(\eta, a_j) C(\eta, a_j)}{V y - \eta^3 V Z_2(x, \eta)} d\eta +$$

$$+ 3a_j \int_{\chi(x)}^y \sqrt{\frac{1}{y - \eta}} R_2(x, \eta) d\eta +$$

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$$\begin{aligned}
& + \frac{a_j^2}{\beta} \int_{\chi(x)}^{\eta} \frac{3\eta + 2\beta^2 x - 2a_j}{V(y-\eta) V Z_2(x, \eta)} C(\eta, a_j) d\eta - a_j f_0 - \\
& - 2a_j \int_{\chi(x)}^{\eta} \frac{(3\eta + 2\beta^2 x) R_1(x, \eta, a_j)}{V(y-\eta)(\eta + a_j^2 x)} d\eta \quad (30)
\end{aligned}$$

$$\begin{aligned}
F_{1,j}^{(1,2)}(x, y, a_j) = & - \frac{1}{2\beta^3} V. p. \int_{\chi(x)}^{\eta} \frac{\eta Q_0^2(\eta, a_j) C(\eta, a_j)}{V(y-\eta)^3 V Z_2(x, \eta)} d\eta + \\
& + \frac{2a_j^2}{\beta^3} \int_{\chi(x)}^{\eta} [Q_0(\eta, a_j) + 2\beta^2 Z_2(x, \eta)] \frac{\eta C(\eta, a_j) d\eta}{V(y-\eta) Z_2(x, \eta)} + \\
& + \frac{a_j}{6} \int_{\chi(x)}^{\eta} (50\eta + 27a_j^2 x) R_2(x, \eta) \sqrt{\frac{\eta}{y-\eta}} d\eta - \\
& - \frac{8a_j}{3} \int_{\chi(x)}^{\eta} (5\eta + 2a_j^2 x) \sqrt{\frac{\eta + a_j^2 x}{y-\eta}} R_1(x, \eta, a_j) d\eta + \\
& + \frac{4a_j^2}{\beta^3} \int_{\chi(x)}^{\eta} \left[Q_0(\eta, a_j) Z_2(x, \eta) + \frac{2}{3} a_j^2 Z_2^2 \right] \sqrt{\frac{Z_2(x, \eta)}{y-\eta}} C(\eta, a_j) d\eta - \\
& - \frac{a_j}{\beta^3} \{ (a_j^2 + \beta^2) f_0 - a_j^2 [e + \chi(x)] f_2 + 2a_j^2 e \chi(x) f_1 \} \quad (31)
\end{aligned}$$

$$\begin{aligned}
L_{1,j}^{(0,1)}(x, y, a_j) = & - \frac{\nabla}{2\beta} V. p. \int_{\chi(x)}^{\eta} \frac{Q_0(\eta, a_j) C(\eta, a_j) d\eta}{V(y-\eta)^2 Z_2(x, \eta)} - \\
& - \frac{\Delta_j}{\beta} \int_{\chi(x)}^{\eta} \frac{C(\eta, a_j) d\eta}{V(y-\eta) V Z_2(x, \eta)} - \frac{3\nabla - 2}{\nabla} a_j e f_0 + \\
& + \frac{2\beta^4}{\Delta} Q_2(x, a_j) f_{-1} - \frac{\Delta_j}{a_j \beta} \int_{\chi(x)}^{\eta} \frac{G_1^{(1)}(x, \eta, a_j)}{V(y-\eta)} d\eta + \\
& + \frac{2e\nabla}{2\beta} \int_{\chi(x)}^{\eta} \frac{G_2^{(0)}(x, \eta, a_j) d\eta}{V(y-\eta)} \quad (32)
\end{aligned}$$

$$L_{1,j}^{(0,2)}(x, y, a_j) = - \frac{\nabla^2}{2\beta} V. p. \int_{\chi(x)}^{\eta} \frac{Q_0^2(\eta, a_j) C(\eta, a_j) d\eta}{V(y-\eta)^3 V Z_2(x, \eta)} -$$

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$$\begin{aligned}
& -\frac{2\Delta_j}{\beta^2} \int_{\chi(x)}^y [\beta^2 Q_{1,j} + 2\nabla\Delta_j Z_2(x, \eta)] \frac{C(\eta, a_j) d\eta}{V y - \eta \sqrt{Z_2(x, \eta)}} - \\
& -\frac{4a_j e}{3\nabla} [(3\nabla - 2)\Delta_j J_1 + [(3\nabla - 2)e - \nabla Q_2(x, a_j)] J_0 + \\
& + \frac{2\beta^4}{\Delta_j} Q_2^2(x, a_j) J_{-1}] - \frac{\nabla a_j e}{\beta^2} [\nabla J_1 + a_j^2 e J_0] + \\
& + \frac{4}{\beta} \nabla a_j \Delta_j e \int_{\chi(x)}^y \left\{ I_{-1}^{(1)}(p_s) - \frac{Q_1(x, \eta, a_j)}{2a_j^2 \nabla e} G_1^{(1)}(x, \eta, a_j) - \right. \\
& - \frac{\Delta_j G_1^{(2)}(x, \eta, a_j)}{6\nabla a_j^2 e} \left. \right\} \frac{d\eta}{V y - \eta} - \frac{a_j \nabla e}{2\beta} \int_{\chi(x)}^y \{ 2\Delta_j J_0 - Q_1(x, \eta, a_j) \times \\
& \times G_2^{(0)}(x, y, a_j) - \Delta_j G_2^{(1)}(x, \eta, a_j) \} \frac{d\eta}{V y - \eta} \quad (33)
\end{aligned}$$

$$\begin{aligned}
L_{1,j}^{(0,3)}(x, y, a_j) = & -\frac{\nabla^2}{2\beta^3} V \cdot p \cdot \int_{\chi(x)}^y \frac{Q_0^3(\eta, a_j) C(\eta, a_j)}{V y - \eta^3 \sqrt{Z_2(x, \eta)}} d\eta - \\
& -\frac{\Delta_j}{3\beta^3} \int_{\chi(x)}^y [3\beta^4 Q_1^2(x, \eta, a_j) + 6\beta^2 \Delta_j (2\nabla - 1) Q_1(x, \eta, a_j) Z_2(x, \eta) + \\
& + \Delta_j^2 (4\nabla - 1) Z_2^2(x, \eta)] \frac{C(\eta, a_j) d\eta}{V y - \eta \sqrt{Z_2(x, \eta)}} - \frac{2a_j e}{5\nabla \beta^2} \{ 15\beta^4 + \\
& + 10\beta^2 + 3 \} [\Delta_j^2 J_2 + 2a_j^2 e \Delta_j J_1 + a_j^4 e^2 J_0] - \beta^2 (15\beta^4 - \\
& - 10\beta^2 - 1) Q_2(x, a_j) [\Delta_j J_1 + a_j^2 e J_0] + 4\beta^4 (1 - 5\beta^2) Q_2^2(x, a_j) J_0 + \\
& + \frac{8\beta^4}{\Delta_j} Q_2^3(x, a_j) J_{-1} \} - \frac{a_j e}{\beta^2} \{ \Delta_j^2 (3\nabla - 2) J_2 + 2\Delta_j [2\beta^2 Q_1(x, 0, a_j) - \\
& - \nabla \Delta_j \chi(x)] J_1 + (\beta^2 Q_1(x, 0, a_j) [Q_1(x, 0, a_j) - 2\Delta_j \chi(x)] + \Delta_j^2 \chi(x)) J_0 \} + \\
& + \frac{\Delta_j}{5a_j \beta} \int_{\chi(x)}^y [60\nabla a_j^2 e Q_1(x, \eta, a_j) I_{-1}^{(1)}(p_s) + 20\nabla a_j^2 \Delta_j e I_{-1}^{(2)}(p_s) - \\
& - 15Q_1^2(x, \eta, a_j) G_1^{(1)}(x, \eta, a_j) - 10\Delta_j Q_1(x, \eta, a_j) G_1^{(2)}(x, \eta, a_j) - \\
& - 3\Delta_j^2 G_1^{(3)}(x, \eta, a_j)] \frac{d\eta}{V y - \eta} - \frac{\nabla a_j e}{2\beta} \int_{\chi(x)}^y [4\Delta_j Q_1(x, \eta, a_j) J_0 + \\
& + 4\Delta_j^2 J_1 - Q_1^2(x, \eta, a_j) G_1^{(0)}(x, \eta, a_j) - 2\Delta_j Q_1(x, \eta, a_j) \times
\end{aligned}$$

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$$\times G_2^{(1)}(x, \eta, a_j) - \Delta_j^2 G_2^{(2)}(x, \eta, a_j) \left\} \frac{d\eta}{V y - \eta} \quad (34)$$

$$\begin{aligned} L_{1,j}^{(1,1)}(x, y, a_j) = & -\frac{\nabla^2}{2\beta} V. p. \int_{\chi(x)}^y \frac{\eta Q_0(\eta, a_j) C(\eta, a_j)}{V y - \eta^3 V Z_2(x, \eta)} d\eta - \\ & - \frac{1}{\beta} \int_{\chi(x)}^y \{ [2\beta^2 Q_1(x, \eta, a_j) - a_j^2 \nabla e] + 2(2\nabla - 1) \Delta_j Z_2(x, \eta) \} \times \\ & \times \frac{C(\eta, a_j) d\eta}{V y - \eta V Z_2(x, \eta)} - \frac{2\beta^2 a_j e}{3\Delta_j \nabla} \{ 2(3\nabla - 4) \Delta_j y + (4 - 3\nabla) [2\beta^2 Q_2(x, a_j) - \\ & - a_j^2 e] J_0 \} - \frac{\beta^2}{\Delta_j} Q_2(x, a_j) [4\beta^2 Q_2(x, a_j) - 3a_j^2 \nabla e] J_{-1} + \\ & + \frac{2}{\beta} \nabla a_j e \int_{\chi(x)}^y \left\{ 2\beta^2 I_{-1}^{(1)}(p_2) - \frac{1}{2\nabla e a_j^2} [2\beta^2 Q_1(x, \eta, a_j) - \right. \\ & \left. - a_j^2 \nabla e] G_1^{(1)}(x, \eta, a_j) \right\} \frac{d\eta}{V y - \eta} - \frac{2\beta \Delta_j}{3a_j} \int_{\chi(x)}^y \frac{G_1^{(2)}(x, \eta, a_j)}{V y - \eta} d\eta - \\ & - a_j \nabla e J_1 - \frac{a_j \nabla e}{\beta} \int_{\chi(x)}^y \left\{ \beta^2 I_0 - \frac{1}{2} Q_1(x, \eta, 0) G_2^{(0)}(x, \eta, a_j) - \right. \\ & \left. - \frac{\beta^2}{2} G_2^{(1)}(x, \eta, a_j) \right\} \frac{d\eta}{V y - \eta} \quad (35) \end{aligned}$$

$$\begin{aligned} L_{1,j}^{(1,2)}(x, y, a_j) = & -\frac{\nabla^3}{2\beta^2} V. p. \int_{\chi(x)}^y \frac{\eta Q_0^2(\eta, a_j) C(\eta, a_j) d\eta}{V y - \eta^3 V Z_2(x, \eta)} - \\ & - \frac{1}{\beta^2} \int_{\chi(x)}^y \{ 2(2\nabla - 1) \Delta_j [2\beta^2 Q_1(x, \eta, a_j) + \Delta_j Q_1(x, \eta, 0)] Z_2(x, \eta) + \\ & + \beta^2 [\beta^2 Q_1^2(x, \eta, a_j) + 2\Delta_j Q_1(x, \eta, a_j) Q_1(x, \eta, 0)] + (4\nabla - 1) \times \\ & \times \Delta_j^2 Z_2^2(x, \eta) \} \frac{C(\eta, a_j) d\eta}{V y - \eta V Z_2(x, \eta)} - \frac{2a_j e}{15\nabla \Delta_j} \{ 3(15\beta^4 + 10\beta^2 + \\ & + 3) [\Delta_j^2 J_2 + 2\Delta_j a_j^2 e J_1 + a_j^4 e^2 J_0] + [3\beta^2(10\beta^2 - \\ & - 15\beta^4 + 1) Q_2(x, a_j) - 10\nabla e^2(3\nabla - 2) e] [\Delta_j J_1 + a_j^2 e J_0 - \\ & - 2\beta^2 [6\beta^2(3\nabla - 6) Q_2(x, a_j) + 5a_j^2 \nabla(4 - 3\nabla)] Q_2(x, a_j) J_0 + \end{aligned}$$

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$$\begin{aligned}
& + \frac{4\beta^2}{\Delta_j} [5\gamma e a_j^2 - 6\beta^2 Q_0(x, a_j)] Q_0(x, a_j) J_{-1} \} + \\
& + \frac{\gamma e a_j}{\beta} \int_{\lambda(x)}^1 [4[2\beta^2 Q_1(x, \eta, a_j) + \Delta_j Q_1(x, \eta, 0)] \Pi_1^{(1)}(p_2) + \\
& + 4\beta^2 \Delta_j J_1^{(2)}(p_2)] \frac{d\eta}{V y - \eta} - \frac{1}{a_j \beta} \int_{\lambda(x)}^1 [\beta^2 Q_1^2(x, \eta, a_j) + \\
& + 2\gamma_j Q_1(x, \eta, a_j) Q_1(x, \eta, 0)] G_1^{(1)}(x, \eta, a_j) \frac{d\eta}{V y - \eta} - \\
& - \frac{2\Delta_j}{3a_j \beta} \int_{\lambda(x)}^1 [2\beta^2 Q_1(x, \eta, a_j) + \Delta_j Q_1(x, \eta, 0)] G_1^{(2)}(x, \eta, a_j) \times \\
& \times \frac{d\eta}{V y - \eta} - \frac{3\beta^2 \Delta_j^2}{5a_j} \int_{\lambda(x)}^1 \frac{G_1^{(3)}(x, \eta, a_j)}{V y - \eta} d\eta - \\
& - \frac{a_j \gamma e}{\beta} \int_{\lambda(x)}^1 \{ [\beta^2 Q_1(x, \eta, a_j) + \\
& + \Delta_j Q_1(x, \eta, 0)] I_0 + 2\beta^2 \Delta_j \Pi_0^{(1)} - \frac{1}{2} Q_1(x, \eta, a_j) Q_1(x, \eta, 0) \times \\
& \times G_2^{(0)}(x, \eta, a_j) - \frac{1}{2} [\beta^2 Q_1(x, \eta, a_j) + \Delta_j Q_1(x, \eta, 0)] G_2^{(1)}(x, \eta, a_j) - \\
& - \frac{1}{2} \beta^2 \Delta_j G_2^{(2)}(x, \eta, a_j) \} \frac{d\eta}{V y - \eta} - a_j e \{ \beta^2 \Delta_j (J_2 - 2x J_1 + \\
& + x^2 J_0) + [(1 + a_j^2) \beta^2 + \Delta_j] e (J_1 - x J_0) + (1 + a_j^2) e^2 J_0 + \\
& + \Delta_j (J_2 - x J_1) + \frac{1}{\beta^2} [\Delta_j + \beta^2 (1 + a_j^2)] e J_1 - 2\Delta_j \chi(x) (J_1 - \\
& - x J_0) - \frac{1}{\beta^2} [\Delta_j + \beta^2 (1 + a_j^2)] e \chi(x) J_0 + \frac{\Delta_j}{\beta^2} [J_2 - 2\chi(x) J_1 + \chi^2(x) J_0] \} \quad (36) \\
& H_{1,0}(x, y) = -\frac{3}{4\beta^2} [3J_2 + [3e + \chi(x)] J_1 - e\chi(x) J_0] + \\
& + \frac{3x}{4} \int_{\lambda(x)}^1 \frac{R_2(x, \eta) d\eta}{V(y-\eta)^2} - \frac{1}{2\beta^2} V. p. \int_{\lambda(x)}^1 \sqrt{\frac{\eta(e-\eta)^2}{(y-\eta)^2 Z_0(x, \eta)}} d\eta \quad (37) \\
& H_{2,0}(x, y) = \frac{5}{16\beta^2} [10J_2 + [\chi(x) - 21e] J_1 - [6\chi^2(x) -
\end{aligned}$$

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$$-10\pi\chi(x) - 10\pi^2 J_2 + \chi(x) [3\chi(x) - 5e] e J_0 -$$

$$-\frac{1}{2\beta^4} V. p. \int_{\chi(x)}^y \sqrt{\frac{(e-\eta)^3 \eta}{(y-\eta)^3 Z_2(x, \eta)}} d\eta + \frac{15x^2}{16} \int_{\chi(x)}^y \frac{R_2(x, \eta) d\eta}{\sqrt{\eta(y-\eta)}} \quad (38)$$

$$H_{1,1}(x, y) = -\frac{3}{4\beta^2} \{5J_2 - [3\chi(x) + 5e] J_2 + 3e\chi(x) J_1\} -$$

$$-\frac{1}{2\beta^2} V. p. \int_{\chi(x)}^y \sqrt{\frac{\eta(e-\eta)}{y-\eta}} \frac{d\eta}{\sqrt{Z_2(x, \eta)}} +$$

$$+\frac{9x}{4} \int_{\chi(x)}^y \sqrt{\frac{\eta}{y-\eta}} R_2(x, \eta) d\eta \quad (39)$$

$$H_{0,1}(x, y) = \frac{J_2}{2} - \frac{1}{2} V. p. \int_{\chi(x)}^y \sqrt{\frac{\eta^3(e-\eta)}{(y-\eta)^3 Z_2(x, \eta)}} d\eta +$$

$$+\frac{3}{2} \int_{\chi(x)}^y \sqrt{\frac{\eta}{y-\eta}} R_2(x, \eta) d\eta \quad (40)$$

$$H_{0,2}(x, y) = \frac{1}{2} J_2 - \frac{1}{2} V. p. \int_{\chi(x)}^y \sqrt{\frac{\eta^3(e-\eta)}{(y-\eta)^3 Z_2(x, \eta)}} d\eta +$$

$$+\frac{5}{2} \int_{\chi(x)}^y \sqrt{\frac{\eta^3}{y-\eta}} R_2(x, \eta) d\eta \quad (41)$$

$$N_0(x, y) = -J_0 + 2J_1 - \frac{\gamma}{2} V. p. \int_{\chi(x)}^y \frac{\sqrt{(e-\eta)\eta} d\eta}{\sqrt{(y-\eta)^3 Z_2(x, \eta)}} +$$

$$+\beta \int_{\chi(x)}^y \frac{I_0(x, \eta) d\eta}{\sqrt{y-\eta}} - \frac{1}{4\beta} \int_{\chi(x)}^y \{ [2\beta^2(\eta-x) - (\beta^2-1)e] G_2^{(0)}(x, \eta) +$$

$$+ 2\beta^2 G_2^{(1)}(x, \eta) \} \frac{d\eta}{\sqrt{y-\eta}} \quad (42)$$

$$N_1(x, y) = \frac{1}{2\beta^2} [e^2 J_0 - e(3\gamma+2) J_1 + 4\gamma J_2] -$$

$$-\frac{\gamma}{2\beta^2} V. p. \int_{\chi(x)}^y \frac{(\gamma\eta-e)\sqrt{(e-\eta)\eta} d\eta}{\sqrt{y-\eta}\sqrt{Z_2(x, \eta)}} - \frac{1}{2\beta} \int_{\chi(x)}^y \{ [3(\gamma-2)e -$$

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$$\begin{aligned}
& -8\beta^2\eta] I_0 + 8\beta^2 I_1) \frac{d\eta}{V_{y-\eta}} - \frac{1}{4\beta} \int_{\chi(x)}^{\eta} ([4\beta^2(\eta-x)^2 - \\
& -3(\nabla-2)e(\eta-x) - 2e^2] G_2^{(0)}(x, \eta) + [8\beta^2(\eta-x) - \\
& -3(\nabla-2)e] G_2^{(1)}(x, \eta) + 4\beta^2 G_2^{(2)}(x, \eta)) \frac{d\eta}{V_{y-\eta}} \quad (43) \\
N_2(x, y) = & -\frac{1}{2\beta^4} [e^2 J_0 - 2(3\nabla+1)e^2 J_1 + \nabla(5\nabla+8)e J_2 - \\
& -6\nabla^2 J_3] - \frac{\nabla}{2\beta^4} V. p. \int_{\chi(x)}^{\eta} \frac{(\nabla\eta-e)^2 V_{\eta(e-\eta)}}{V_{y-\eta}^3 V_{Z_2(x, \eta)}} d\eta - \\
& -\frac{1}{\beta} \int_{\chi(x)}^{\eta} ([2e^2 + 5(\nabla-2)e\eta - 9\beta^2\eta^2] I_0 + [18\beta^2\eta - \\
& -5(\nabla-2)e] I_1 - 9\beta^2 I_2) \frac{d\eta}{V_{y-\eta}} - \frac{1}{4\beta} \int_{\chi(x)}^{\eta} ([6\beta^2(\eta-x)^2 - \\
& -5(\nabla-2)e(\eta-x)^2 - 4e^2(\eta-x)] G_2^{(0)}(x, \eta) + \\
& + 2[9\beta^2(\eta-x)^2 - 5(\nabla-2)e(\eta-x) - 2e^2] G_2^{(1)}(x, \eta) + \\
& + [18\beta^2(\eta-x) - 5(\nabla-2)e] G_2^{(2)}(x, \eta) + 6\beta^2 G_2^{(3)}(x, \eta)) \frac{d\eta}{V_{y-\eta}} \quad (44)
\end{aligned}$$

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In formulas (17)-(44) are introduced the following designations:

$$\begin{aligned}
J_k &= \int_{\chi(x)}^{\eta} \frac{\eta^k d\eta}{V(y-\eta) \eta(e-\eta)(\eta+\beta^2 x-e)} \\
J_{-1} &= \int_{\chi(x)}^{\eta} \frac{d\eta}{\left(-\frac{a_j^2 e}{\Delta_j} - \eta\right) V(y-\eta) \eta(e-\eta)(\eta+\beta^2 x-e)} \\
G_1^{(m)}(x, \eta, a_j) &= \frac{2a_j^2 \nabla e}{\nabla_j} f_{-2}^{(m)}(p_2) - (a_j^2 + \beta^2) f_{-1}^{(m)}(p_2) + \\
&+ a_j^2 f_{-1}^{(m)}(p_1) + \beta^2 f_{-1}^{(m)}(p_2) \quad (m=1, 2, 3) \\
G_1^{(m)}(x, \eta, a_j) &= f_{-1}^{(m)}(p_1) + f_{-1}^{(m)}(p_2) \quad (m=0, 1, 2)
\end{aligned}$$

$$p_1 = \eta - e, \quad p_2 = \frac{e}{\beta^2} + \eta, \quad p_3 = \frac{Q_1(0, \eta, a_j)}{\Delta_j}$$

$$I_m = \int_{\tilde{\gamma}(\eta)}^x \frac{\xi^m d\xi}{\sqrt{(x-\xi)(e+\xi-\eta)\left(\frac{e}{\beta^2} + \eta - \xi\right)}}$$

$$I_m^{(p)} = \int_{\tilde{\gamma}(\eta)}^x \frac{(x-\xi)^m d\xi}{(p-\xi)^4 \sqrt{(x-\xi)(e+\xi-\eta)\left(\frac{e}{\beta^2} + \eta - \xi\right)}}$$

In the last/latter integral p , are taken values p_1 , p_2 and p_3 . All integrals of types I_m , $I_m^{(p)}$, J_k and J_{-1} are expressed as elliptical integrals

$$I_0 = \frac{2\beta}{\sqrt{\nabla e}} E(\delta, q)$$

$$I_1 = \frac{2}{\beta\sqrt{\nabla e}} [-Q_1(x, \eta, 0) \Pi(\delta, q^2, q) + Q_1(0, \eta, 0) E(\delta, q)]$$

$$I_2 = -\frac{2\eta}{3\beta^2} \sqrt{\eta(e-\eta) Z_2(x, \eta)} - \frac{4Q_1(x, \eta, 0)}{3\beta^2 \sqrt{\nabla e}} [e(1-\beta^2) + \beta^2 x +$$

$$+ 2\beta^2 \eta] \Pi(\delta, q^2, q) + \frac{2}{3\beta^2 \sqrt{\nabla e}} [2Q_1^2(0, \eta, 0) + \beta^2(x + \eta - e) Q_1(0, \eta, 0) + \beta^4(e - \eta)x] E(\delta, q)$$

$$I_1^{(p)}(p_2) = -\frac{2\beta\Delta_j^2 Q_1(x, \eta, 0)}{a_j^2 \sqrt{\nabla e^3} Q_1(x, \eta, a_j)} \Pi\left(\delta, \frac{a_j^2 e \nabla q^2}{\beta^2 Q_1(x, \eta, a_j)}, q\right) +$$

$$+ \frac{2\beta^3 \Delta_j}{a_j^2 \sqrt{\nabla e}} E(\delta, q)$$

$$I_2^{(p)}(p_2) = -\frac{1}{a(x, \eta, a_j)} \left[\frac{Q_1(x, \eta, 0)}{\sqrt{\nabla}} \Pi(\delta, q^2, q) + \frac{a_j^2 \sqrt{\nabla}}{\beta \Delta_j} E(\delta, q) + \right.$$

$$+ \frac{\beta \Delta_j^2 Q_1(x, \eta, 0) b(x, \eta, a_j)}{a_j^2 \sqrt{\nabla^3} Q_1(x, \eta, a_j)} \Pi\left(\delta, \frac{a_j^2 \nabla e q^2}{\beta^2 Q_1(x, \eta, a_j)}, q\right) -$$

$$\left. - \frac{\beta^3 \Delta_j}{a_j^2 \sqrt{\nabla^3}} b(x, \eta, a_j) E(\delta, q) + \frac{\Delta_j \sqrt{\eta(e-\eta) Z_2(x, \eta)}}{\beta Q_0(\eta, a_j)} \right]$$

$$J_0 = \frac{2}{\beta \sqrt{xy}} E\left(\frac{\pi}{2}, q_1\right)$$

$$J_1 = \frac{2}{\beta \sqrt{xy}} \left\{ (y - e) \Pi\left(\frac{\pi}{2}, x, q_1\right) + e E\left(\frac{\pi}{2}, q_1\right) \right\}$$

$$J_2 = \frac{2}{\beta \sqrt{xy}} \left\{ \chi(x)(e - y) \Pi\left(\frac{\pi}{2}, \frac{e x}{y}, q_1\right) + (Z_2 + e)(e - y) \times \right.$$

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$$\begin{aligned}
 & \times \Pi\left(\frac{\pi}{2}, x, q_1\right) + [\beta^2 x(e-y) - 2e] E\left(\frac{\pi}{2}, q_1\right) \Big\} \\
 J_0 = & -\frac{1}{2} \frac{ey\chi(x)}{\beta\sqrt{xy}} E\left(\frac{\pi}{2}, q_1\right) - \frac{3\chi(x)}{4\beta\sqrt{xy}} (Z_0 + e) \left[(e-y) \times \right. \\
 & \left. \times \Pi\left(\frac{\pi}{2}, \frac{ex}{y}, q_1\right) + y E\left(\frac{\pi}{2}, q_1\right) \right] \\
 J_{-1} = & \frac{2\Delta_f(e-y)}{\beta^2 Q_0(y, \alpha_j)} \Pi\left(\frac{\pi}{2}, \frac{e\beta^2 x}{Q_0(y, \alpha_j)}, q_1\right) + \frac{2}{\beta^2} E\left(\frac{\pi}{2}, q_1\right)
 \end{aligned}$$

where they are designated

$$\begin{aligned}
 \delta &= \arcsin \sqrt{\frac{e(\eta + \beta^2 x - e)}{\beta^2 \eta(x - \eta + e)}}, \quad q = \sqrt{\frac{\beta^2(x - \eta + e)}{ve}} \\
 x &= \frac{y + \beta^2 x - e}{\beta^2 x}, \quad q_1 = \sqrt{\frac{e(y + \beta^2 x - e)}{\beta^2 xy}} \\
 a(x, \eta, \alpha_j) &= \frac{1}{\beta^2} (\beta^2 p_3^2(\eta, \alpha_j) - [Q_1(0, \eta, 0) + \beta^2(x + \\
 & + \eta - e)] p_3^2(\eta, \alpha_j) + [Q_1(0, \eta, 0)(x + \eta - e) - \beta^2(e - \eta)x] p_2(\eta, \alpha_j) + \\
 & + (e - \eta) Q_1(0, \eta, 0)x) \\
 b(x, \eta, \alpha_j) &= \frac{1}{\beta^2} (3\beta^2 p_3^2(\eta, \alpha_j) - 2[Q_1(0, \eta, 0) + \beta^2(x + \\
 & + \eta - e)] p_3(\eta, \alpha_j) + Q_1(0, \eta, 0)(x + \eta - e) - \beta^2(e - \eta)x)
 \end{aligned}$$

Through E and P are designated the elliptical integrals respectively of the first and third kinds.

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For a region σ_1 (Fig. 1) the formula of the coefficients rotary

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**NON-STATIONARY DOWNWASH BEHIND A TRIANGULAR
WING IN SUPERSONIC MOTION**

R. Sh. SOLOMONIAN

S u m m a r y

Rated formulae for non-stationary downwash behind a thin slightly curved wing which has a triangular formula in the plan and supersonic edges whose apex is turned forward when it moves in the ideal compressible fluid with supersonic speed are given in this paper.

These formulae are obtained for area points situated on the plane of the wing between the disturbance waves, their reflection from the back edge and the back edge itself at small Strouhal numbers.

The above formulae make it possible to continue the downstream calculations at infinitum according to the formulae previously derived for this part of the wing plane.

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